



Research Article

Analyzing Ocular Diseases Through Persistence Diagrams and Topological Data Analysis

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Received: 20/12/2025, Accepted: 27/02/2026, Published(online): 31/05/2026

Abstract

This study explores the topological characteristics of affected eyes through the framework of Topological Data Analysis (TDA), with a focus on Betti numbers derived from persistence diagrams. A dataset comprising retinal images of both normal and disease affected eyes was analyzed using Ripser, a tool for computing persistent homology in grayscale images. By extracting Betti numbers (β_0 and β_1) from the persistence diagrams, we quantitatively captured the topological features such as connected components and loops present in each image. Comparative analysis revealed notable differences in the distribution of Betti numbers between normal and disease eyes, indicating measurable alterations in the topological structure of disease affected eyes. These findings highlight the potential of TDA as a non-invasive and quantitative method for distinguishing ocular health conditions based on topological invariants.

Keywords: Bettinnumbers; Connected components; Diabetic Retinopathy

1 Introduction and Preliminaries

TDA uses tools from topology to extract meaningful patterns, shapes, or structures from complex datasets. Rather than relying only on numerical summaries or statistical assumptions, TDA explores the geometric and topological structure of the data. To study the topology of a dataset, we often start by converting point cloud data into a simplicial complex, which is a network of points, edges, triangles, and higher-dimensional analogs that capture connectivity.

Definition 1.1. Let $V = \{v_0, v_1, \dots, v_k\} \subset \mathbb{R}^d$ be a set of points, and let $d, k \in \mathbb{N}$. Any sum of the kind

$$\sum_{i=0}^k a_i v_i, \quad \text{with} \quad \sum_{i=0}^k a_i = 1$$

is their **affine combination**. The set of all affine combinations of V is called the **affine hull** of V .

Definition 1.2. Let $k, d \in \{0, 1, \dots\}$ with $k \leq d$. The convex hull of an affinely independent family $V = \{v_0, v_1, \dots, v_k\} \subset \mathbb{R}^d$ is a geometric k -simplex σ in $\mathbb{R}^d$, i.e., $\sigma = \text{Conv}(V)$.

Definition 1.3. Let $d \in \{0, 1, \dots\}$. A (finite) set of geometric simplices is called a **geometric simplicial complex** $K \subset \mathbb{R}^d$ if and only if:

b: If $\sigma, \tau \in K$, then $\sigma \cap \tau$ is either empty or a common face of both

a: If $\sigma \in K$ and τ is a face of σ , then $\tau \in K$

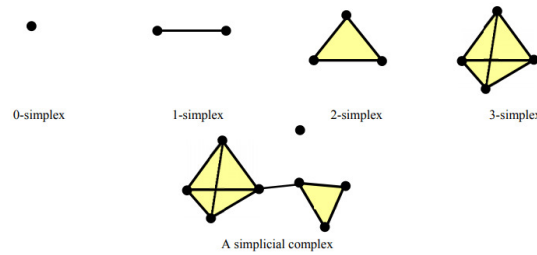


Fig 1: Geometric simplicial complex

Definition 1.4. A **triangulation** of a subspace $X \subset \mathbb{R}^d$ is a simplicial complex K in $\mathbb{R}^d$ such that $|K| \cong X$, where $d \in \{0, 1, \dots\}$.

Note: The body of K [notation: $|K|$] is the union of all simplices of K .

Definition 1.5. Let X be a metric space, and let $S \subset X$ be a finite subset. Select a scale such that $r \geq 0$. An abstract simplicial complex, the Rips complex $\text{Rips}(S, r)$ is defined as follows:

1. The vertex set is S .
2. A subset $\sigma \subseteq S$ is a simplex if and only if $\text{Diam}(\sigma) \leq r$.

Definition 1.6. A p -chain is a formal sum $\sum_{i=1}^{n_p} \lambda_i \sigma_i^p$, where σ_i^p is an orientated simplex of dimension p in K for each i and $\lambda_i \in F$. This formalism takes orientation into account: if σ is an orientated simplex, the simplex σ with the opposite orientation is $(-1) \cdot \sigma = -\text{sigma}$.

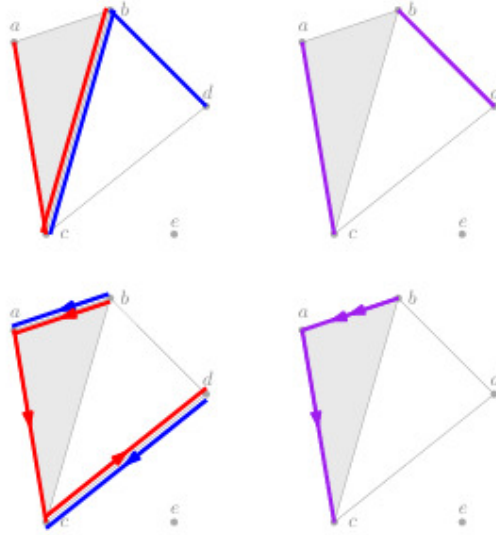


Fig 2: Example for p-chain

Definition 1.7. The **chain group** $C_p(K; F)$ is the vector space of all p -chains.

Definition 1.8. The boundary map represented by ∂_p can be defined as the mapping from $C_p(K, F) \rightarrow C_{p-1}(K, F)$ given $p \in N$.

Remark 1.9. Composition $\partial \circ \partial$ is the constant function zero.

Definition 1.10. The **chain complex** is the set of chain groups connected by the border mappings. $\cdots \xrightarrow{\partial} C_n(K; F) \xrightarrow{\partial} C_{n-1}(K; F) \rightarrow C_{n-2}(K; F) \xrightarrow{\partial} \cdots \xrightarrow{\partial}$

Definition 1.11. Let K be an abstract simplicial complex. Select a field (F) and $(q \in 0, 1, \dots)$. We specify:

- the group of q -cycles as $Z_q(K; F) = \ker \partial_q \subset C_q(K; F)$,
- the group of q -boundaries as $B_q(K; F) = \text{Im } \partial_{q+1} \subset Z_q(K; F) \subset C_q(K; F)$,
- the group of q -homology group as the quotient $H_q(K; F) = Z_q(K; F) / B_q(K; F)$.

Remark 1.12. Betti numbers are invariant under homology classes and are used to study topological structures and Betti numbers of dimension k are denoted by β_k

2 Computing Persistence Diagram and Betti Number of Eye in Different Condition

Betti numbers serve as fundamental topological invariants that quantify the intrinsic structure of a space by capturing its connectivity and the presence of holes in different dimensions. Specifically, β_0 represents the number of connected components, describing 0-dimensional holes; β_1 denotes the number of one-dimensional holes or loops, which can be interpreted as circular voids in the usual R^2 sense; and β_2 measures the number of two-dimensional voids or enclosed cavities within the space. Together, these numbers provide a concise summary of the space's topological complexity. In this paper we use 3D Ripser software for calculating persistence diagram and betti number of normal and affected eye. We collect the data set from kaggle.com and persistence diagram of normal eye and affected eye are analysed. A detailed comparison of the Betti numbers corresponding to normal and diabetic retinopathy-affected retinal images is provided in the following tables.

Betti Number
(14620, 9669)
(22810, 16508)
(15463, 11093)
(9817, 5915)
(12259, 6726)
(3187, 1861)
(5301, 3115)
(2730, 1343)
(10614, 5146)

Table 1

Betti Number
(37196, 22371)
(39406, 21265)
(38707, 21771)
(39546, 21740)
(1146, 857)
(1570, 1092)
(1477, 1075)
(1157, 731)
(10033, 8176)

Table 2

The Betti numbers corresponding to normal eye images generally lie above one lakh. From above table, a significant reduction in Betti numbers can be observed for disease affected eyes. This decrease may be attributed to the degeneration or loss of complex structural features in the retinal tissue caused by the disease.

3 Conclusion

This study demonstrates the effectiveness of persistent homology in distinguishing between normal and affected eye images. The above table of betti numbers of affected eyes shows a drastic difference from that of normal eyes. For normal eye both the betti numbers β_0 and β_1 lies approximately above one lakh. In this case complex network structures results in high betti number values but in the case of disease affected eye damage of blood vessels cause death of some simplices resulting in low betti numbers. Thus in this comparative study of normal and affected eyes persistent diagram and betti numbers can be used as an efficient tool.

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